

# Peirce's Triadic Logic: Modality and Continuity

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## 1 Introduction

In 1909, from around January 7th to February 23, Charles Sanders conducted the first experiments with three valued logic ('triadic logic' in his terminology) by generalizing the matrix method for defining truth functions from two to three values.<sup>1</sup> These experiments predate the pioneering work on three and many valued logic by Jan Łukasiewicz (1920, translated in 1970) and Emil Post (1921) by about 10 years. While Peirce's work on the subject is no where near formally advanced as Łukasiewicz's and Post's, it nonetheless interesting to see just how far his anticipations go. Many of the formal aspects of Peirce's project have been explored by Atwell Turquette. He has shown how the operators Peirce defines can be used to construct a variety of different three valued logics, all of which are maximally strong and functionally complete (Turquette, 1969)[209]. Turquette has also discovered a complex network of dualistic and trimorphic relations between these operators (Turquette, 1972). Each one of Peirce's three valued truth functions is the dual of another with respect to the typical interpretation of negation, as well as a trimorph of two other operators with respect to a unary operator Peirce defines that is not seen again until Post's work. It is unclear how

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<sup>1</sup>A method that Peirce himself pioneered (Anellis, 2012).

much of this Peirce was aware of, but it is striking how powerful his fragmentary work really is.<sup>2</sup>

Formal developments aside, one aspect of Peirce's triadic logic that remains especially unclear is the motivations behind it. Philosophers and logicians have explored three and many valued logics for a plethora of reasons, including concerns about future contingent propositions as well as vague predicates and propositions. These concerns do not seem to be precisely what Peirce had in mind while he was working towards his own system. To date, there have been two accounts that attempt to pin down Peirce's philosophical motivations, one due jointly to Max Fisch and Turquette (1966), and another due to Robert Lane (1999). Almost all of the direct evidence is contained entirely on a single page in Peirce's Notebook. There are really only two threads to follow on said page: One involving Peirce's notion of "Modes of Being," another stemming from a peculiar example involving an ink blot.

Fisch and Turquette, drawing on Peirce's Modes of Being, claim that Peirce appears to have been concerned with modality. Lane, on the other hand, claims that the propositions motivating triadic logic are non-modal and that Peirce's inkblot example illustrates that he was really motivated by his concerns for continuity and continua. It might seem like we have two dissonant fragments of explanation as to why Peirce invented his triadic logic. While it may be difficult to see how these two fragments could be united, I will argue that in fact they can. Contrary to Lane, throughout this essay I will argue that we cannot properly understand the philosophical concerns behind Peirce's triadic logic without reference to both his views on modality as well as continuity. It is clear that at some point Peirce came to think of continuity in modal terms. I will argue further that Peirce developed his triadic logic to accommodate

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<sup>2</sup>For more discussion of the formal aspects of Peirce's triadic logic notes, see [Turquette \(1967, 1968, 1976, 1978, 1981, 1988\)](#).

modality and continuity with a mind towards capturing an aspect of another philosophical theory of his: his hypothetical cosmology. He wanted a logic to capture the evolving universe as he saw it. I will do this by re-examining Fisch and Turquette’s account as well as Lane’s, and illustrate the difficulties each face in presenting a holistic picture of what Peirce was up to. Fisch and Turquette, and Lane following them, only draw on three pages from Peirce’s Logic Notebook (MS 339), however there are three additional connected pages that can and should be considered. A full description of these pages would be beyond the scope of this paper, however, in making my argument I will draw on the title of one of these pages in particular.

In section 1, I recount Fisch and Turquette’s findings. In section 2, I turn to Lane’s account and argue against his claim that modality did not factor into Peirce’s reasoning. In section 3, I unite these two views and show how they coincide with Peirce’s hypothetical cosmology.

## 2 Modality: Fisch and Turquette’s Account

The first mention of Peirce’s triadic logic outside of his notebook pages seems to have been in an article Turquette produced for an edited volume entitled *Studies in the Philosophy of Charles Sanders Peirce* (Turquette, 1964). While giving a full description of these pages is beyond the scope of this essay, it will be useful at the outset to describe the single page that contains nearly all of the textual evidence we have to go off of in determining Peirce’s motivations. This is the final page in the cluster devoted to triadic logic. The page is titled “Triadic Logic” and is dated February 23, 1909.

The text begins:

“Triadic logic is that logic which, though not rejecting entirely the principle of excluded middle, nevertheless recognizes that every propo-

sition, S is P, is either true or false, or else S has a lower mode of being such that it can neither be determinately P, nor determinately not P, but is at the limit between P and not P.”

Peirce does not explain here what he means by ‘mode of being’ but this notion figures importantly in his philosophy elsewhere. In 1903, he writes: “My view is that there are three modes of being. I hold that we can directly observe them in elements of whatever is at any time before the mind in any way. They are the being of positive qualitative possibility, the being of actual fact, and the being of law that will govern facts in the future” (CP 1.23). It is already clear that modality was built into this notion.

The note goes on to say “Of course it remains true, as far as the principle of contradiction is concerned that the state of things represented by the proposition cannot be  $V$  and  $F$ , *verum atque falsum* and must be  $V + F$  if by  $F$  is meant  $L + F$  ( $L$  signifying the limit, i.e. that S is not capable of the determination P or of the determination  $F$ ).”<sup>3</sup> Peirce follows this up with a rather strange example of the kinds of propositions he is concerned with: “Thus, a blot is made on a sheet. Then every point of the sheet is unblackened or blackened. But there are points on the boundary line; and those points are insusceptible of being blackened or of being unblackened, since these predicates refer to the area about S and a line has no area about any point of it.”

At first glance this seems to be an example of a category mistake. The proposition ‘point  $x$  on the boundary line is blackened’ should apparently take the value  $L$ , because  $x$  would have to have area in order to be blackened or unblackened, and points on lines do not have area. Thus, this predicate is undefined for  $x$ . The strange thing is that points themselves do not have any area, which runs contrary to Peirce’s claim that “every point of the sheet is

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<sup>3</sup>When Peirce uses  $+$  in this way, it is just a notational variant of  $\vee$ .

blackened or unblackened.” It is unclear how this example is supposed to relate to Peirce’s earlier comment about modes of being. As far as examples go, this one is not very enlightening. It turns out that the example actually pertains to Peirce’s theory of continuity, as Lane argues (1999).

At the bottom of the page we find Peirce triumphantly declaring “Triadic logic is universally true. But Dyadic Logic is not absolutely false, it is only *L*.” So it seems Peirce was trying to extend dyadic logic rather than completely revise it.

Fisch and Turquette begin their account by discussing the various operators and formal developments on three of the six pages connected to triadic logic. They note that on the formal side, Peirce seems to have been motivated by a concern for duality and functional completeness when he defined his operators. But when it comes to his philosophical reasons, Fisch and Turquette are much more tentative. They offer two possibilities: 1) that Peirce was motivated by considerations of modality, as Łukasiewicz was, 2) that his motivations were due to his doctrine of “Tychism,” which basically holds that there is fundamental indeterminacy in the world.

They begin their explanation stating that “It is clearly indicated that the motivation arises from problems associated with the kind of proposition which ‘has a lower mode of being such that it can neither be determinately *P*, nor determinately not-*P*’ — assuming that the proposition in question is of the form *S* is *P*” (Fisch and Turquette, 1966, 77). They draw this evidence from the page I have provided a description of, [seq. 645<sup>4</sup>](#), where Peirce states as much himself, and claim that this is enough to suggest that he was motivated by concerns for modality, as Łukasiewicz was. More specifically, they claim he was interested in modal issues in which a third truth value seems necessary to

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<sup>4</sup>I will henceforth refer to manuscript pages from MS 339 by their order in the sequence of pages digitally reproduced on the Harvard Mirador Website

evaluate propositions, such as future contingents. The most famous example is Aristotle's future sea battle case. If I pronounce 'there will be a sea battle tomorrow', at the time of my utterance it is unclear what the truth value of the proposition is. Whether or not there does happen to be a sea battle the next day, it may still seem inappropriate to say the proposition is true or false, and we may want to reserve an intermediate value for the proposition. Nonetheless, Peirce's one remark on [seq. 645](#) seems rather thin evidence to justify the claim that it was these kinds of modal propositions he was hoping to capture.

To further support their claim, Fisch and Turquette draw our attention an entry in *The Prescott Book* (MS 277) from January 1908. The passage discusses modality in connection with "potentiality", "actuality", and "necessitation" ([Fisch and Turquette, 1966](#)). It reads:

*Potentiality* is the absence of Determination (in the usual broad sense) not of a mere negative kind but a positive capacity to be Yea or Nay; not ignorance but a state of being...

*Actuality* is the Act which determines the merely possible...

*Necessitation* is the support of Actuality by reason..."

They then go on to cite a passage concerning the three universes in the Logic Notebook from August 1908 ([seq. 550](#)). On that page Peirce writes:

"I have been for some months working out a Hypothetical Cosmology of both worlds, the here and the hereafter... Now, I say that the co-reality of the three universes, 1st of Ideas, 2nd of Occurrences (existent things and actual events), 3rd of powers to bring two substances into relation to each other (and I will call powers of this sort Reasons), must accordingly, be supposed capable of rational explanation."

This provides evidence that Peirce was thinking about modality throughout the year leading up to his experiments in triadic logic. It also provides a possible explanation of what Peirce meant when he writes of lower modes of being on [seq. 645](#). Peirce’s comments in all of these passages seem to imply a commitment to a hierarchy of modes of being, and it certainly makes sense to think of “potentiality” being at the bottom and “necessitation” on the top if he really was concerned with modality.

There is other textual evidence linking the universes Peirce mentions to his triadic logic. The page in which Peirce defines his three valued operators, [640](#), is a *verso*. On the *recto*, [seq. 639](#), the title makes explicit reference to these universes. It reads: “Studies of Modal, Temporal, and Other Logical Forms which Relate to Special Universes.” The rest of the page contains a brief discussion about Boole’s system of logic that is not obviously related to the title. He discusses a “fundamental quadratic”<sup>5</sup> that appears to be connected to a rather mysterious unary function, symbolized by  $X$  (Chi), that figures prominently in the other pages. While the discussion on that page is not entirely helpful, the title shows that these universes were on his mind when he was experimenting with triadic logic. Fisch and Turquette do not mention this page, but if I am correct that there is a connection between the “special universes” in the title of [seq. 639](#) and the “three universes” Peirce discussed in the notebook previously, then this would be strong evidence in favor of Fisch and Turquette’s view that he was operating with some notion of modality in mind.

Peirce continued thinking and writing about modality in triadic terms throughout the final years of his life and it seems fairly clear that when he did so he had this kind of hierarchy in mind. In an unpublished essay cited by Fisch and Turquette, entitled *The Art of Reasoning Elucidated*, he writes:

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<sup>5</sup>Which he writes as  $x(x - 1) = 0$  or  $(x - V)(x - F) = 0$ .

“Now, in this respect, a simply assertory proposition differs just half as much from the assertion of a Possibility, or that of a Necessity, as these two differ from each other. For, as we have seen above, that which characterizes and defines an assertion of Possibility is its emancipation from the Principle of Contradiction, while it remains subject to the Principle of Excluded Third; while that which characterizes and defines an assertion of Necessity is that it remains subject to the Principle of Contradiction, but throws off the yoke of the Principle of Excluded Third; and what characterizes and defines an assertion of Actuality, or simple Existence, is that it acknowledges allegiance to both formulae, and is thus just midway between the two rational “Modals”, as the modified forms are called by all the old logicians.”  
(MS 678, 1910)

Notice that when Peirce writes of “Actuality” he locates it between the possible and the necessary, suggesting a hierarchical ordering. Some of what is contained in this passage is perfectly consistent with the way we currently think of modal propositions too. While we would not normally say that the law of non-contradiction does not apply to modals, it is trivial to say that for any possible proposition  $P$ , ‘it is possible that  $P$  and it is possible that not  $P$ .’ This is likely what Peirce means when he claims potentials are emancipated from this principle. We can say something similar with regard to necessities and the principle of excluded middle. While we would not say that PEM fails for necessary statements, we also would not say of any proposition  $P$  that ‘it is necessary that  $P$  or it is necessary that not  $P$ ’ since it could be the case that neither. This is likely what Peirce meant when discussing these principles, although admittedly, this is not the standard way of understanding either of them.



Fisch and Turquette also use these notions to clarify what Peirce says on [seq. 645](#). They claim “Essentially, Peirce seems to be saying that triadic logic may be interpreted as a modal logic which is designed to deal with the indeterminacies resulting from that mode of being which Peirce has called ‘Potentiality’ and ‘Real Possibility’” (Fisch and Turquette, 1966). This is why on that page Peirce claims that dyadic logic is “not universally false it is only *L*.” He may be saying that dyadic logic is limited because it fails to account for these real indeterminacies.

This brings us to their second point about Peirce’s possible motivations: his Tychism. Unlike other philosophers working on logic around the time these pages were written, Peirce was committed to the view that there is fundamental indeterminacy in the world that cannot be removed by rendering propositions less ambiguous.<sup>6</sup> For Peirce, this indeterminacy is metaphysical in nature, and does not necessarily result merely from a lack of knowledge or the language we use. This follows from his 1898 definition of Tychism, as “the doctrine that absolute chance is a factor in the universe” (CP 6.201). Contrary to some authors of his time, like Russell, Peirce did not believe that “the universe is [necessarily] regulated by law down to every detail” (Ibid). He was not a determinist, and thought that some facts of the world might come down to arbitrary chance. A little less than two weeks after he completed [seq. 645](#), Peirce writes in a letter to William James “I hold to my ‘tychism’ more than ever.” Fisch and Turquette use this evidence to connect Peirce to others who have held a similar belief in some kind of irreducible indeterminacy, like Hans Reichenbach or Werner Heisenberg, both of whom were motivated by undecidable statements about quantum mechanics. They further remark that Gödel’s undecidable statements in mathematics could be another example of this kind of indeterminacy. Obviously Peirce could not have been aware of any of these developments, however his

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<sup>6</sup>Unlike Bertrand Russell who in 1906 seems to have held that indeterminacy can always be removed by carrying a propositions determination further (Russell, 1906).

tychism might be seen as anticipating these kinds of results.

So it seems that Peirce's introduction of the value ' $L$ ' to logic may have been a way of importing his tychism to logic. This would be consistent with complaints Peirce made about the "oldfashioned logicians"<sup>7</sup> in a draft of the just mentioned letter, written just three days after seq. 645. According to Fisch and Turquette's reproduction, it reads:

"I have long felt that it is a serious defect in existing logic that it takes no heed of the *limit* between two realms. I do not say that the Principle of Excluded Middle is downright *false*; but I *do* say that in every field of thought whatsoever there is an intermediate ground between *positive assertion* and *positive negation* which is just as Real as they. Mathematicians always recognize this, and seek for that limit as the presumable lair of powerful concepts; while metaphysicians and oldfashioned logicians, — the sheep [and] goat separators, — never recognize this. The recognition does not involve any denial of existing logic, but it involves a great addition to it."

Is Peirce making reference to his triadic logic here? His remarks about this "Real intermediate ground" may be driven by his commitment to his tychism. Furthermore, his remarks here that acknowledging such a middle ground "does not involve any denial of existing logic, but it involves a great addition to it" seem awfully similar to his remarks about the difference between dyadic and triadic logic on [seq. 645](#). Thus, it seems Peirce's three-valued efforts might be an attempt to insert his tychism into logic. This explanation of Peirce's motivation is not necessarily distinct from the possibility that he was motivated by modal considerations, but it may help elucidate his understanding of modality.

Fisch and Turquette seem to make a strong case that in experimenting with

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<sup>7</sup>Presumably, these are more conservative logicians who were resistant to the advances made in logic by Boole, De Morgan, and Peirce, in favor of traditional syllogistic logic.

three-valued logic, Peirce was motivated by concerns for his doctrine of tychism, which more broadly factors into his understanding of modality. At this point it seems likely that Peirce, like Łukasiewicz, thought it necessary to deviate from classical logic in order to deal with certain kinds of modal propositions. This connection is strengthened by considering some of what is written on the pages Fisch and Turquette neglect to mention. However, this suggestion runs contrary to the view Lane takes on Peirce’s experiments. There is one major defect with Fisch and Turquette’s account and this is that they pay no special attention to Peirce’s ink blot example on [seq. 645](#). The only mention of it is in a sentence that declares it “is not very helpful in providing an answer” ([Fisch and Turquette, 1966](#)). Lane’s account explores this example in much greater detail.

### 3 Continuity: Lane’s Account

Writing more than 30 years after Fisch and Turquette, Robert Lane expresses a very different view of what Peirce was up to with his triadic logic. On Lane’s view, Peirce was motivated not by concern for modality, but by his understanding of continuity and the continuum. In this section I will elaborate on Lane’s account.

Lane’s account makes two claims about Peirce’s motivations: one negative, one positive. The negative claim is that the kinds of propositions motivating triadic logic are non-modal. The positive claim is that Peirce was truly motivated by his views on continuity. Lane’s route to his denial that Peirce was concerned modality begins with an exposition of Peirce’s views on the principle of excluded middle (PEM) and the principle of non-contradiction (PC). His account here is an expansion of a previous paper about Peirce’s views on PEM and PC ([Lane, 1999](#)).

Lane takes his negative claim to follow from the fact that Peirce’s rejection of PEM does not actually imply a failure of bivalence. However, removing modality

entirely from our account would amount to ignoring about half of the limited evidence available. How would we explain Peirce's use of the phrase "modes of being" which he certainly understood in modal terms? Why would he have put modality in the title of [seq. 639](#) if modality had nothing to do with his project? For these reasons I think we think we should reject Lane's negative point and continue to entertain the possibility that modality was a motivating factor for triadic logic.

The issues with Lane's negative claim by no means undermine the value of his positive claim. Continuity was a major concern in Peirce's philosophy throughout the entirety of his life, and especially in this later period. He came to it not only through his own philosophical lens, but also from reading other influential figures, like Cantor, Dedekind, and Bolzano. Lane comes to this contribution by exploring an aspect of [seq. 645](#) that Fisch and Turquette almost entirely neglect: his inkblot example.

While Peirce's example initially appears to be concerned with cases of predication involving category mistakes, Lane's analysis reveals that it is more likely about breaches of continuity. Lane claims that the subject being Considered here is the boundary line,  $B$ . The proposition ' $B$  is black' then would relieve the valuation  $L$  because it is not true that  $B$  is black, nor that  $B$  is not. Rather, Lane claims quoting Peirce,  $B$  "is at the limit between [black] and non-black" (Lane, 1999).  $B$  in this example, constitutes a breach of continuity. It interrupts at the continuous space on the sheet of paper, and lacks the properties possessed by the regions on either side. Given the importance Peirce placed on continuity in his philosophy, this is a much more natural interpretation of the example than category mistakes.

It might seem odd that Peirce would place so much importance on such a narrow range of propositions that he thought it necessary to revise logic to

accommodate them, but Peirce thought continuity was of the utmost importance.

To illustrate this importance, Lane draws our attention to this passage:

“[T]he idea of continuity, or unbrokenness. . . plays a great part in all scientific thought, and the greater the more scientific that thought is; and it is the master key which adepts tell us unlocks the arcana of philosophy” (CP 1.163).

Peirce’s emphasis on continuity in philosophy and in science stems from another doctrine that he endorsed, not altogether distinct from his tychism. He called this synechism, and says it is the “doctrine that all that exists is continuous” (CP 1.172). His reasons for endorsing synechism are far beyond the scope of this thesis, but he likely perceived the continuity of time, space, and thought as evidence supporting his endorsement.

There is further textual evidence to support the link between the inkblot example and Peirce’s views on continuity. He used a similar example in the eighth of his Cambridge Conference lectures in 1898 (CCL, all of which are presented in [Peirce](#)), which Lane quotes, where he is quite clearly talking about continuity and boundary properties:

“I draw a chalk line on the board. . . what I have really drawn there is an oval line. For this white chalk-mark is not a line, it is a plane figure in Euclid’s sense—a surface, and the only line there is the line which forms the limit between the black surface and the white surface. . . the boundary between the black and white is neither black; nor white, nor neither, nor both” (CP. 6.203).

The inkblot example seems to be a brief expression of the ideas Peirce was elaborating here. Thus, continuity must have factored into the motivations behind triadic logic. While it is apparent continuity was part of the picture; it

is not at all clear why this would call for a revision of logic. To help answer this question, Lane tracks the evolution of Peirce's understanding of continuity and his definition of continua across four stages throughout the course of his life (Lane, 1999).

Peirce's understanding around the time of the triadic logic notes is most clearly stated in a marginal note in Peirce's personal copy of *The Century Dictionary* (CP 6.166-168). Here he tells us that his previous understanding resulted from a misunderstanding that Kant himself fell into: "He himself and I after him, understood that to mean infinite divisibility, which plainly is not what constitutes continuity since the series of rational fractional values is infinitely divisible but is not by anybody regarded as continuous" (CP 6.168). Thus, it seems infinite divisibility is not itself enough to make a series continuous. The reason Peirce mentions the series of "rational fractional values" here is because between every pair of these there are real and rational values that are not part of the series. In the marginal note Peirce goes on to say, "[t]he precise definition is still in doubt; but Kant's definition, that a continuum is that of which every part has itself parts of the same kind, seems to be correct." (Ibid). This, however, Peirce insists must be understood differently from the mistaken interpretation of continuity as infinite divisibility. In the same passage Peirce tells us that a true continuum wouldn't actually contain any points at all: "a line, for example, contains no points until the continuity is broken by marking the points. In accordance with this it seems necessary to say that a continuum, where it is continuous and unbroken, contains no definite parts; that its parts are created in the act of defining them and the precise definition of them breaks the continuity" (Ibid). It is not clear why Peirce thought this but it will be explored in the following section. However, the understanding expressed here seems to have persisted relatively unchanged until the end of his life, as Lane claims.

At this point it might seem like we have two dissonant fragments of explanation as to why Peirce invented his triadic logic. On the one hand, his comments about “modes of being” on [seq. 645](#) seem to be connected to modality, albeit in an idiosyncratic way, as Fisch and Turquette suggest. On the other, his inkblot example on the same page seems to be connected to his views about continua while having nothing to do with modality, as Lane contends. It might be difficult to see how these fragments could be united, but I argue that, in fact, they can be. At some point it is clear that Peirce came to think of continuity in modal terms.

In 1898, in his Cambridge lectures, he states:

“a continuum is a collection of so vast a multitude that in the whole universe of possibility there is not room for them to retain their distinct identities; but they become welded into one another. Thus, the continuum is all that is possible, in whatever dimension<sup>8</sup> it be continuous” (NEM 4: 343).

Notice here Peirce’s mention of “the universe of possibility.” This is likely connected to the special Universes mentioned on [seq. 639](#) and to the three Universes he discusses earlier in his notebook, on [seq. 550](#).

There are other examples of Peirce using modal phrases in connection with continuity. The chalkboard version of the inkblot example also makes use of these. Just prior to the portion Lane quotes, Peirce writes:

“Let the clean blackboard be a sort of diagram of the original vague potentiality, or at any rate of some early stage of its determination.

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<sup>8</sup>“Dimension” here and in the next quoted passage may be a technical term. This logical notion is only used by Peirce and a handful of other Boolean logicians (Dipert, 1994). In the logic of relations, Peirce and his students sometimes thought of propositions as two dimensional. Roughly, these dimensions are the objects propositions range over or are true for and the “times” at which it is true (Ibid). Peirce disliked the use of ‘time’ here and preferred to speak of “possible situations” (Ibid).

This is something more than a figure of speech; for after all continuity is generality. . . This blackboard is a continuum of two dimensions, while that which it stands for is a continuum of some indefinite multitude of dimensions. This blackboard is a continuum of possible points; while that is a continuum of possible dimensions of quality, or is a continuum of possible dimensions of a continuum of possible dimensions of quality, or something of that sort” (CP 6.203).

In the lecture, Peirce is using the chalkboard example to illustrate his hypothetical cosmology as based on his synechism. Given the modal undertones of his cosmology, this establishes a link between modality and the inkblot example. There is one final passage I would like to draw attention to in regards to the connection between modality and continuity, one that Lane himself cites. It comes from an unfinished essay entitled “A Sketch of Logical critic” written in 1911 for inclusion in a collection intended to honour Lady Welby:

“Personally, I agree entirely with James against Dedekind’s view; and hold that there would be no actually existent points in an existent continuum, and that if a point were placed in a continuum it would constitute a breach of the continuity. Of course, there is a possible, or potential, point-place wherever a point might be placed; but that which only maybe is necessarily thereby indefinite, and as such, and in so far, and in those respects, as it is such, it is not subject to the principle of contradiction, just as the negation of a may-be, which is of course a must be, (I mean that if ‘*S* may be *P*’ is untrue, then ‘*S* must be non-*P*’ is true), in those respects in which it is such, is not subject to the principle of excluded middle” (CP 6:182).

This passage pretty clearly to establishes a connection between Peirce’s thoughts on triadic modality and the continuum. He goes on to remark that “logic here



seems to touch metaphysics” (Ibid). From these examples we can see that these notions were connected in Peirce’s thought for at least 13 years.

Lane’s paper makes important strides in explaining the connection between Peirce’s triadic logic and his views on continuity. However, his claim that Peirce’s motivations excluded modality is mistaken.<sup>9</sup> It is clear now that modality, triadic logic, and continua were all connected in Peirce’s mind. In the next section, I will flesh out the details of this connection.

## 4 Modality, Continuity, and Hypothetical Cosmology

So far I have shown that viewing Peirce’s motivations for triadic logic as due exclusively to either modality or continuity is mistaken. These two notions were intimately connected for him. In this section show how modality and continuity meet under Peirce’s hypothetical cosmology. I will do this by answering a handful of questions more specific in scope than the broad question of what was motivating Peirce. These are: 1. What was Peirce trying to formalize or bring into the scope of logic? 2. Why did he need his third truth-value, L, to do this? And 3. What propositions take the value L?

To answer these questions, I will draw mainly from an unpublished manuscript entitled “A Fourth Curiosity” (AFC), which was written in the same year Peirce conducted his three valued experiments, as well as CCL. The former was originally intended for publication in *The Monist* as part of a series Peirce was writing called “Amazing Mazes” (Peirce, 1908). A handful of these were published but AFC and another paper called “A Third Curiosity” were not.

AFC is an odd document in part because of its wide range of subject matter

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<sup>9</sup>Some have even attributed to him “a modal logical view of set theory” (Putnam, 1995).

and also because it is not altogether clear what the point of it is. It begins with discussion of the logic of relations, then moves on to philosophical and mathematical notions of time, then modes of being, before finally ending with discussions of cardinality and infinity, making reference to Cantor, Dedekind, and Bolzano. Many of these are topics that have come up in our previous discussion. My hope is that, given that Peirce may have been writing this document at more or less the same time as he wrote of triadic logic in his notebook, his explication of these similar topics will provide insight into what he meant in his notes.

My discussion will rely heavily on Peirce’s “special universes,” which are clearly analogues of his “modes of being.” These are both related to his three categories, some version of which was endorsed as far back as 1867, when he argues that these categories are necessary to give a unified concept of experience.

#### **4.1 What was Peirce trying to represent?**

One observation that can be gleaned from an examination of the triadic notebook pages is that Peirce seemed to have an idea of what he wanted to extend logic to represent, before he knew exactly how to do so. This is evident from the change of approaches he makes in that cluster of pages from his logic notebook (seq. 637–645). He begins by adding a new operator to his existential graphs, then switches back to a traditional symbolic notation, and alternates between two and three-valued semantics before he seems to have decided on seq. 645 that adding a truth value was the correct approach. So, what was Peirce trying to represent? Furthermore, why was classical logic incapable of representing it?

Part of the reason classical logic is not up to the task has to do with the universe it represents. In AFC, Peirce tells us that his previous work on logic has focused on “existential relations” which have only to do with one logical universe:

“An *existential* relation is distinguished from others by two marks. In the first place, its different subsets all belong to one universe. . . In the second place, an existential relation or relationship differs from some other relations and relationships in a respect which may be described in two ways, according as we employ collective or distributive forms of expression and thought” (CP 6.318).

This second point of difference is more easily understood when it is expressed according to this collective form.

“Speaking collectively, the one logical universe, to which all the correlates of an existential relationship belong, is ultimately composed of *units*, or subjects, none of which is in any sense separable into parts that are members of the same universe. For example, no relation between lapses of time—say, between the age of Agamemnon and that of Homer—can be an existential relation, if we conceive every lapse of time to be made up of lapses of time, so that there are no indivisible units of time” (Ibid).

Perhaps it is helpful to frame Peirce’s discussion here within the special universes of his hypothetical cosmology. Recall that Peirce conceived of three special universes, of ideas, occurrences, and powers or reasons. He also identified these universes with his modes of being, claiming the mode of being of the first universe is possibility, the second actuality, and the third necessity (seq. 552). The relations that Peirce is discussing here are located within the second universe, which is actual. According to him, this universe is made up of units, and this means it is not continuous, so these existential relations must be non-continuous as well.

In the following paragraph, Peirce goes on to state that his previous work on logic has been limited to the study of existential relations:

“My reasons for mostly limiting the scope of my logical studies of relations were, firstly, that these are very tangible and logically tractable; secondly that the great body of other sorts of relations differ from these merely in being indeterminate in some respects in which existential relations or some species of these are determinate, so that the logical theory of these virtually puts the student into possession of the logical theory of all but a very few recondite relations. . .” (CP 6.319).

Since the bulk of Peirce’s previous logical work is concerned with what we now call classical logic, we can infer that this is the subject he is drawing limits around. Classical logic deals with what he calls existential relations. So, on Peirce’s view, classical logic represents a limited class of relations.

From this, and Peirce’s remarks about special universes and modes of being in his notebook, I think we can answer the questions posed at the beginning of this section. Why was classical logic inadequate for Peirce? Based on these passages, it seems he thought it was only capable of representing relations in one of his “special universes.” More specifically, it is only capable of representing his second Universe, “of occurrences (existent things and actual events)” (MS 339, seq. 550). So, it seems that in conducting his three-valued experiments, Peirce was trying to extend logic so that it could adequately represent the other two universes, which are modal in nature. There are a couple of reasons given here for why classical logic was not up for this task. One is that it treats relations that are indeterminate as determinate (CP 6.319). The second has to do with classical logics subject being “ultimately composed of *units*” (CP 6.318). Because of this, classical logic is apparently incapable of representing relations that are continuous, as is evident from Peirce’s time example. Because classical logic was apparently incapable of expressing continuous relations, this also meant it

was incapable of representing universes other than the second.<sup>10</sup> This is why Peirce says, on seq. 645, that dyadic (or classical two-valued) logic is not false or incorrect, only that it is limited.

A similar point could be made regarding Peirce's "modes of being," which correspond to his universes. Classical logic, being restricted "to the existential class" of relations, must also be restricted to the existential mode of being, which Peirce characterizes as follows: "In the metaphysical sense, existence, is that mode of being which assists in the genuine dyadic relation of a strict individual with all the other such individuals of the same universe" (CP 6.336). Here Peirce explicitly connects his "modes of being" to his logical universes. Given his characterization of existential relations at the beginning of AFC, as well as his comment that most of his work on logic has been restricted to those relations, we can infer that on his view classical logic would have been restricted to the existential mode of being as well.

Peirce's discussion of the other modes of being will help us determine what is missing, given this restriction. He writes:

"so, then, there are these three modes of being: first, the being of a feeling, in itself, unattached to any subject, which is merely as atmospheric possibility. . . ; secondly, there is the being that consists in arbitrary brute action upon other things. . . and thirdly, there is living intelligence from which our reality and power are derived: which is rational necessity and necessitation" (CP 6.342).

He elaborates on these further:

"A feeling is what it is, positively, regardless of anything else. Its being is in it alone, and it is a mere potentiality. A brute force, as,

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<sup>10</sup>This does not necessarily mean that Peirce thought classical logic incapable of expressing truths about continuous *domains*. This restriction appears to only apply to relations.

for example, an existent particle, on the other hand, is nothing for itself; . . . its being is actual, consists in action, is dyadic. That is what I call existence. A reason has its being in bringing other things into connexion with each other; its essence is to compose: it is triadic, and it alone has a real power” (CP 6.343).

So, Peirce identifies the existential mode of being with the second of these. Then, what is missing if classical logic is restricted to this mode, are the other two. Consequently, the reason classical logic was not up to the task is that it is incapable of representing potentiality or necessity.<sup>11</sup>

I believe here we have an answer to the first question. In his notebook pages Peirce was attempting to extend logic to be capable of representing features of his other two universes, or his other two modes of being. These seem to be connected to alethic modalities in some way. However, to say that he was trying to create a simple modal logic like we are familiar with today would be an oversimplification. Peirce had already created a modal logic by this point, by adding a modal operator to his beta graphs, without resorting to abandoning bivalence (CP 4.510–529 and 573–584).<sup>12</sup> Nevertheless, some clues might be found by examining his comments about the limitations of existential relations. Specifically, his comment that the logical universe these relations pertain to “is ultimately composed of units” and his time example which would seem to illustrate existential relations cannot be continuous. Given that classical logic seems to be restricted to this class of relations this might explain why Peirce

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<sup>11</sup>In “A Fourth Curiosity,” Peirce also speaks to the ranking that is implied by his remarks in the first paragraph of seq. 645. He explains what he means by a “lower mode of being,” but this is quite opaque. He writes: “This much, however, seems clear about such existence: namely that there ought to be two grades of it; a lower kind, approximating to the inner being of a simple quality, yet existential, instead of being merely potential, consisting in the action of a thing upon all the other things of the same universe, and measuring by its intensity its remoteness from each of them. A whole universe of such existents can only have the lower, or internal grade of existence” (CP 6.346).

<sup>12</sup>J. Zeman has proven in a 1964 [unpublished doctoral thesis](#) that Peirce’s modal graphs are equivalent to S4 and S5 [Zeman \(1964\)](#).

says that dyadic logic “is not absolutely false, it is only  $L$ ” (seq. 645). It is limited because it cannot capture continuity.

## 4.2 Why did Peirce need $L$ ? What propositions are $L$ ?

These questions are more difficult to answer. Since the continuum of reals is the most obvious example of continuity, we might expect that there would be propositions about the real numbers that would take the value  $L$ , at least according to his notion of a continuum. Sadly, it is difficult to imagine what these mathematical examples would be or even if there are any. The only obvious examples of  $L$ -propositions seem to be the boundary propositions that Lane has identified from the inkblot example. Even though Lane has helpfully connected this passage to Peirce’s views on continuity, it still is not obvious why this required a third value. Luckily, Peirce used this kind of example to illustrate points about continuity more than just the couple of times we have seen so far and some of these are much more illustrative of the philosophical significance of boundary propositions. It appears that Peirce may have had a slightly different continuum in mind when he gave the inkblot example, and he was thinking more along the lines of time rather than the reals.

In his paper, “The Law of Mind”, published in *The Monist* 1892, Peirce makes use of this kind of example again:

“Suppose a surface to be part red and part blue; so that every point on it is either red or blue, and, of course, no part can be both red and blue. What, then, is the color of the boundary line between the red and the blue? The answer is that red or blue, to exist at all, must be spread over a surface; and the color of the surface is the color of the surface in the immediate neighborhood of the point. I purposely use a vague form of expression. Now, as the parts of the

surface in the immediate neighborhood of any ordinary point upon a curved boundary are half of them red and half blue, it follows that the boundary is half red and half blue. In like manner, we find it necessary to hold that consciousness essentially occupies time; and what is present to the mind at any ordinary instant is what is present during a moment in which that instant occurs. Thus, the present is half past and half to come.”

This passage, not cited by Lane, is clearly relevant to how this example is used in the notes on triadic logic. Here Peirce gives a bit more indication as to why the boundary point is neither blue nor red (or blackened or unblackened in the familiar version on seq. 645) and consequently the need for  $L$ . It is apparently because that property belongs to a point based on what is immediately next to it and on a boundary line this would take on both of the properties on either side. When the properties on either side are contradictory, as with black and not black, then it seems reasonable to say that propositions attributing either property to points on the boundary line would have to be  $L$  because they would have both in some sense.

Peirce’s connecting this example to time is helpful to see why he thought these kinds of propositions were significant. When the continuum is time, it appears he thought the present acted as the boundary line between the past and the future. So presumably there are propositions about the present that would rightly be assigned the value  $L$ . If Peirce was thinking about continuity and time, that would explain the inclusion of the word ‘temporal’ in the title of the seq. 639. But what kind of temporal propositions would take the value  $L$ ? One common feature in all the various versions of this example is that on either side of the boundary there is a property present that the other side lacks. These properties are usually such that a particular object cannot have both. For



Peirce, when the past and future are what is separated, the property possessed on one side but lacking on the other is that of determinacy. On his view, the past is wholly determinate while the future is indeterminate. Following through with the other examples, a proposition asserting that the present has either of these properties would be assigned the value  $L$ .

To spell this out further and show why Peirce would have thought this significant, it will be helpful to return to the Cambridge lecture in which he gave the chalkboard version of the boundary example. In the context of the lecture, Peirce was using the example to illustrate his hypothetical cosmology.

“Now continuity is shown by the logic of relations to be nothing but a higher type of that which we know as generality. It is relational generality.<sup>13</sup>

How then can a continuum have been derived? Has it for example been put together? Have the separated points become welded, or what?

Looking upon the course of logic as a whole we see that it proceeds from the question to the answer — from the vague to the definite. And so likewise all the evolution we know of proceeds from the vague to the definite. The indeterminate future becomes the irrevocable past... However it may be in special cases, then, we must suppose that as a rule the continuum has been derived from a more general continuum, a continuum of higher generality” (CP 6.190–191).

Here it seems Peirce is trying to draw inferences about continua in general based on special cases. Time is the second of these. Peirce’s remarks about the past

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<sup>13</sup>CCL seems to be the first place where Peirce associates continuity with relational generality. By generality of the implied lower type, he means something like *universals* (Moore, 2007). Continuity then is a higher type of generality than universality. It is not clear why he came to this view or exactly what he means by statements such as these. It might be generality over properties rather than individuals.

and future above seem to confirm what I have just said in trying to apply his boundary example to time. This process of going from “the vague to the definite” appears to apply to more continua than time as well. All of this is part of Peirce’s broad cosmological or metaphysical theory.

“From this point of view we must suppose that the existing universe, with all its arbitrary secondness, is an offshoot from, or an arbitrary determination of, a world of ideas, a Platonic world; . . .

If this be correct, we cannot suppose the process of derivation, a process which extends from before time and from before logic, we cannot suppose that it began elsewhere than in the utter vagueness of completely undetermined and dimensionless potentiality.

The evolutionary process is, therefore, not a mere evolution of the existing universe, but rather a process by which the very Platonic forms themselves have become or are becoming developed” (CP 6.192–194).

This passage exposes Peirce’s realism. He appears to believe that this aspect of his continua that is more easily understood in temporal form also applies in metaphysics. Just as the future is determined when it becomes the past, it appears Peirce thought something similar is happening with forms as the universe evolves. The forms receive an arbitrary determination in “the existing universe.” It seems as though Peirce thought just about everything is involved with this kind of a process in one way or another. In the lecture, he uses these notions to describe his hypothetical cosmology:

“We shall naturally suppose, of course, that existence is a stage of evolution. This existence is presumably but a special existence. We need not suppose that every form needs for its evolution to emerge into this world, but only that it needs to enter into some theatre of

reactions, of which this is one.

The evolution of forms begins or, at any rate, has for an early stage of it, a vague potentiality; and that either is or is followed by a continuum of forms having a multitude of dimensions too great for the individual dimensions to be distinct. It must be by a contraction of the vagueness of that potentiality of everything in general, but of nothing in particular, that the world of forms comes about” (CP 6.195-196).

What Peirce is attempting to describe is how the universe evolves, which involves some movement from the indeterminate to the determinate. He also thought that this applied beyond our universe, to “the whole Platonic world” as well (CP 6.200). One topic discussed in the previous section comes up again in this passage. He says the evolution he is talking about begins with a vague potentiality that consists in “a continuum of forms having a multitude of dimensions too great for” its individuals to be distinct. As we have seen before, this probably means that the forms he spoke of are uncountably infinite. Peirce seems to attribute this property to potentiality whenever he discusses it and this is probably because of the vast number of ways things could unfold according to his cosmological theory of evolution. This passage also can help us to understand why he was so interested in boundaries. The boundary lines in Peirce’s examples must be analogues of the “theatre of reactions” he mentions. This is where “the vagueness of that potentiality of everything in general, but of nothing in particular” contracts as he puts it. The boundary is where potentials begin to become determined. In the case of time, the present is the boundary at which the indeterminate future begins to become the determinate past.

Against this background, the blackboard example that Lane cites will make a lot more sense and help us understand how all of this worked in Peirce’s mind.

“Let the clean blackboard be a sort of diagram of the original vague potentiality, or at any rate of some early stage of its determination. This is something more than a figure of speech; for after all continuity is generality. This blackboard is a continuum of two dimensions, while that which it stands for is a continuum of some indefinite multitude of dimensions. This blackboard is a continuum of possible points; while that is a continuum of possible dimensions of quality, or is a continuum of possible dimensions of a continuum of possible dimensions of quality, or something of that sort. There are no points on this blackboard. There are no dimensions in that continuum. I draw a chalk line on the board. This discontinuity is one of those brute acts by which alone the original vagueness could have made a step towards definiteness. There is a certain element of continuity in this line. Where did this continuity come from? It is nothing but the original continuity of the blackboard which makes everything upon it continuous. What I have really drawn there is an oval line. For this white chalk-mark is not a line, it is a plane figure in Euclid’s sense — a surface, and the only line there, is the line which forms the limit between the black surface and the white surface. Thus the discontinuity can only be produced upon that blackboard by the reaction between two continuous surfaces into which it is separated, the white surface and the black surface. The whiteness is a Firstness — a springing up of something new. But the boundary between the black and white is neither black, nor white, nor neither, nor both. It is the pairedness of the two. It is for the white the active Secondness of the black; for the black the active Secondness of the white” (CP 6.203, CCL).

It is now apparent that Peirce was using this example to illustrate his cosmological metaphysics. The blank board is the “vague potentiality” he mentions earlier in the lecture. The chalk line is a continuity breach that renders this original potentiality more definite. The boundary lines are the “theatre of reactions” where the separation between the two becomes determined. It is a “second” to the two because it exists only in its relation to them.

This understanding of Peirce’s boundary examples makes it easier to come up with propositions of the sort he likely imagined would take the value  $L$ . The value  $L$  still seems to be reserved for boundary propositions however, this might not be as narrow a range of propositions as previously thought. Returning to temporal examples, we might think that whenever some event happens there is always some exact moment at which it occurs. This moment is the boundary between the “indeterminate future” (which Peirce would also call a vague potentiality) and the “irrevocable past.” It is the moment that determines the actual event from the mere possibility of it. It is a second to each side because it is defined by them. Following Peirce’s discussion above, the boundary would have to be partially indeterminate and determinate. Now, because both sides of the boundary are continuous (and possibly because it is partially indeterminate), the boundary itself must be continuous also. Consider the proposition ‘ $x$  was born at  $t$ .’ Because time is continuous, we can always carry this determination further, say by asking whether  $x$  was born at  $t.1$  or  $t.001$ , and so forth. Thus, this is a kind of boundary proposition in these regards and would likely be the kind of proposition.

## 5 Conclusion

Above I believe I have established that Peirce had considerations of both modality and continuity in mind while he was conducting his experiments with triadic logic.

It is apparent that Peirce was trying to take logic to a higher level of generality. He wanted to elevate logic to capture his metaphysical theories by extending it to the “Universe of Possibility” and the “Universe of Necessity”. Because he thought the three universes were related in a continuous process, he needed to account for the indeterminacy involved to extend logic this way. And this is because there is inherent fuzziness at the boundary between the determinate and indeterminate. Peirce was motivated by a desire to extend logic to facilitate reasoning about the evolving universe represented by his hypothetical cosmology.

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